



Coordination in the use of money



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ABSTRACT

Fundamental models of money, while explicit about the frictions that render money essential, are silent on how agents actually coordinate in its use. This paper studies this coordination problem, providing an endogenous map between the primitives of the environment and the beliefs on the acceptability of money. We show that an increase in the frequency of trade meetings, besides its direct impact on payoffs, facilitates coordination. In particular, for a large enough frequency of trade meetings, agents always coordinate in the use of money.

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1. Introduction

The principle that the use of money should be explained by its essentiality is well established among monetary theorists.¹ Indeed, a precise description of the frictions (e.g., limited commitment and limited record-keeping) which render money essential is a central element in fundamental models of money, such as random matching, overlapping generations, and turnpike models. However, these models typically exhibit multiple equilibria, including one in which money is not valued and have no say on which equilibrium will be played. This paper takes steps towards filling this gap by exploring how the primitives of an economy impact agents' ability to coordinate in the use of money.

The analysis is cast in a search model of money along the lines of [Kiyotaki and Wright \(1993\)](#). The key departure from their environment is the assumption that money is not fiat. We let the economy experience different states over time, and assume that while money is intrinsically useless in a large region, some states have an impact on the characteristics of money. There are states where money is either intrinsically useful or convertible into something useful, and states where either the use of money may involve some intrinsic disutility or money becomes less valuable as a medium of exchange (which is the case, for instance, of a hyperinflation). In particular, there are faraway states where either accepting or not accepting money is a strictly dominant action.

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¹ Money is essential if it achieves socially desirable allocations which could not be achieved otherwise. [Kocherlakota \(1998\)](#) and [Wallace \(2001\)](#) are key references.

The main result is that there exists a unique rationalizable equilibrium. If the gains from trade are relatively large and agents are relatively patient, agents coordinate in the use of money in all states in which money has no intrinsic utility (and also in some states in which money has negative intrinsic utility). If instead the gains from trade are small and agents are relatively impatient, agents never coordinate in the use of money in states in which money has no intrinsic utility (and also in some states in which money brings positive intrinsic utility).

It turns out that patience, which can also be interpreted as the frequency of trade meetings, critically helps agents coordinate in the use of money – more so than an increase in the gains from trade in a given meeting. Intuitively, an agent that expects to face many trading opportunities is willing to accept money even if he holds relatively pessimistic beliefs as to the acceptance of money in the near future. Since all agents make the same reasoning, pessimistic beliefs on the acceptability of money cannot be part of an equilibrium. As the frequency of trade meetings goes to infinity, agents coordinate in the use of money irrespective of the difference between the utility from consumption and the production cost.

The paper relates to the literature on global games and to the literature on equilibrium selection in dynamic games with complete information, in particular, [Frankel and Pauzner \(2000\)](#) (FP), and [Burdzy et al. \(2001\)](#) (BFP). We share with these papers both the assumption that there exist faraway states where it is strictly dominant to choose a particular action and the result that the ensuing equilibrium is unique. One relevant difference is that in a monetary economy, the benefit of exerting effort in exchange for money depends on how agents will behave in the near future and not on how they behave today. This eliminates equilibria in which an agent chooses a particular action in a given period simply because he believes that all the other agents will choose the same action in that period. In order to deal with this problem, FP and BFP assume that each agent has only a small chance of changing his action in any given period, which prevents the multiplicity of equilibria that arises when agents are allowed to continuously shift from one action to another. We do not need an extra assumption to tackle this issue.² In terms of results, an important difference is that in FP and BFP, an increase in the time discount factor does not help to select the efficient outcome. In their model, if the time discount factor is large enough, the risk-dominant equilibrium is selected regardless of whether it is efficient.³ This suggests that the coordination problem involved in the use of money is markedly different from that present in other (non-monetary) settings in which coordination matters.

Finally, there is a strand of models within monetary economics that studies how the addition of an intrinsic utility to money may help to reduce the set of equilibria. In overlapping generations models, the focus is on the elimination of monetary equilibria that exhibit inflationary paths (e.g., [Brock and Scheinkman, 1980](#); [Scheinkman, 1980](#)). In search models of money, the objective is to characterize the set of fiat money equilibria that are limits of commodity-money equilibria when the intrinsic utility of money converges to zero (e.g., [Zhou, 2003](#); [Zhu, 2003, 2005](#); [Wallace and Zhu, 2004](#)). A result that comes out of this work is that if goods are perfectly divisible and the marginal utility is large at zero consumption, autarky is not the limit of any commodity money equilibria. This result critically depends on the assumption of a sufficiently high probability that the economy reaches a state where money acquires an intrinsic utility. In contrast, our results on coordination in the use of money hold even if the probability that money ever acquires intrinsic utility is arbitrarily small and the probability it ever acquires intrinsic disutility is equal to one. Thus, coordination in the use of money is not simply the result of working backwards from a distant future where money is sure to acquire a positive intrinsic utility. What matters to determine whether agents will coordinate or not are the primitives of the environment, i.e., the gains from trade and the rate at which agents meet.

The paper is organized as follows. In [Section 2](#), we present the model, deliver the main result, and discuss the assumptions. We also examine how the primitives of the environment impact agents' ability to coordinate in the use of money. In [Section 3](#), we conclude. All omitted proofs are available in an online appendix.

2. The main result

In this section, we present the environment of the model and shows it exhibits a unique rationalizable equilibrium.

2.1. Environment

The environment is a version of [Kiyotaki and Wright \(1993\)](#). Time is discrete and indexed by t . There are k indivisible and perishable goods and the economy is populated by a unit continuum of agents uniformly distributed across k types. A type i agent derives utility u per unit of consumption of good i and is able to produce one unit of good $i+1$ (modulo k) per period, at a cost $c < u$. Agents maximize expected discounted utility with a discount factor $\beta \in (0, 1)$. There are k distinct sectors, each sector specialized in the exchange of one good. In every period, agents choose which sector they want to visit but inside a sector they are randomly and pairwise matched. Each agent faces one meeting per period, and meetings are independent across agents and over time. There exists a storable and indivisible object, which we call money. An agent can hold at most one unit of money at a time, and money is initially distributed to a measure m of agents.

² A similar issue arises in static coordination games. The literature on global games (e.g., [Carlsson and van Damme, 1993](#); [Morris and Shin, 1998](#)) has shown that a small amount of asymmetric information can lead to a unique equilibrium. In contrast, there is symmetric information in our model but that does not lead to multiplicity of equilibria.

³ Analogously, in global games, the risk-dominant equilibrium is selected if the idiosyncratic differences in information among agents are arbitrarily small.

In any given period, the economy is in some state $z \in \mathbb{R}$. The economy starts at $z=0$ and the state changes according to a random process $z_{t+1} = z_t + \Delta z_t$, where Δz_t follows a continuous probability distribution that is independent of z and t , with expected value $E(\Delta z)$ and variance $\text{Var}(\Delta z) > 0$. The state of the economy does not affect the preferences for goods and the production technology. Thus in the absence of money, states could be interpreted as realizations of a sunspot variable. However, the state of the economy affects the characteristics of money if z is either sufficiently large or sufficiently small. If an agent holds one unit of money at the beginning of a period in a state $z > \hat{z}$, he can choose between bringing this unit to a trading sector, obtaining an intrinsic flow utility zero; or keeping this unit throughout the period, obtaining an intrinsic flow utility $(1 - \beta)\gamma(z) > 0$. In turn, if an agent holds one unit of money at the beginning of a period in a state $z < -\hat{z}$, he can choose between bringing this unit to a trading sector, obtaining an intrinsic flow utility $\gamma(z) < 0$; or keeping this unit throughout the period, obtaining an intrinsic flow utility zero. Money provides no intrinsic utility, i.e., $\gamma(z) = 0$, in states $z \in [-\hat{z}, \hat{z}]$. We assume that $\gamma(z)$ is weakly increasing everywhere and strictly increasing whenever $\gamma(z) \neq 0$. Moreover, $\lim_{z \rightarrow \infty} \gamma(z) = \gamma$, where $\beta\gamma > c$; and $\lim_{z \rightarrow -\infty} \gamma(z) = \phi$, where $-\phi > u$. Throughout, we think of \hat{z} as a very large number.

We assume that $(1 - m)u + mc > \gamma$. This implies that the net benefit of bringing money to the trading sector which offers the good one likes, under the belief that money is going to be accepted as a medium of exchange, is larger than the intrinsic utility of money. Therefore, as it will be shown, even though the possibility that money may acquire some intrinsic utility affects society's ability to coordinate in its use, from an individual perspective, the benefit of money comes solely from its use as a medium of exchange.

2.2. Interpreting the dominance regions

The environment departs from [Kiyotaki and Wright \(1993\)](#) in the description of the characteristics of money. [Kiyotaki and Wright \(1993\)](#) and, for that matter, any standard search model of money assume that money is fiat, i.e., it is intrinsically useless and inconvertible. Following [Wallace \(1980\)](#), intrinsic uselessness means that money is *never* wanted for its own sake, while inconvertibility means that the issuer of money, if there is one, will *never* stand ready to convert money into something desirable for its own sake. In our environment, since $\gamma(z) \neq 0$ in some states, money is not fiat. We offer below two alternative interpretations to the function $\gamma(z)$, one which relaxes the intrinsic uselessness of fiat money and another which relaxes the inconvertibility of fiat money.

The assumption that money may be intrinsically useful in some states of the world captures the idea that the physical properties of the object used as money may not be invariant over time. For example, the state z may reflect technological developments which allow us to convert money into a shinier object, say a jewel. In this case, if an agent holds one unit of money at the beginning of a period in a state $z > \hat{z}$, he can use the technology to convert money into a jewel, in which case he obtains a flow utility $(1 - \beta)((z - \hat{z})/(1 + z - \hat{z}))\gamma$, where $\beta\gamma > c$. As a result, if z is sufficiently large, it is strictly dominant to produce in exchange for money, irrespective of the behavior of other agents. In another direction, the state z may reflect changes in the environment which increase the transaction costs of using money as a medium of exchange, or which make it more costly to carry money into a sector which produces the good one likes. In this case, if an agent chooses to bring money into a trading sector in a state $z < -\hat{z}$, he obtains a flow utility $\gamma(z) = (|z - \hat{z}|/(1 + |z - \hat{z}|))\phi$, where $-\phi > u$. Hence if z is sufficiently small, an agent is not willing to produce in exchange for money. Under this interpretation of $\gamma(z)$, we have $\gamma(z) = p(z)\gamma$ for $z > \hat{z}$, $\gamma(z) = 0$ for $z \in [-\hat{z}, \hat{z}]$, and $\gamma(z) = p(z)\phi$ for $z < -\hat{z}$, where $p(z) = |z - \hat{z}|/(1 + |z - \hat{z}|)$.

The assumption that money is inconvertible, although common in models of money with explicit microfoundations, is quite restrictive. [Wallace \(1980\)](#), for instance, points out that fiat money systems are rare throughout history. More recently, arguments in favor of the gold standard have been treated with less skepticism in the wake of the great recession of 2009. For example, Thomas Hoenig, the current president of the Federal Reserve Bank of Kansas City, argued that “the gold standard is a very legitimate monetary system.”⁴ Alan Greenspan, the former Chairman of the Federal Reserve System, said that “there are numbers of us, myself included, who strongly believe that we did very well in the 1870 to 1914 period with an international gold standard.”⁵ Admittedly, they were not arguing for the reestablishment of convertibility of paper money into gold, but we view those statements as highlighting that convertibility, although unlikely, is not an outright impossibility.

The state z can thus be interpreted as capturing the commitment of the money issuer to the value of money. In states $z \in [-\hat{z}, \hat{z}]$, the money issuer does not interfere in the economy. However, in states $z > \hat{z}$, there is a probability $p(z) = (z - \hat{z})/(1 + z - \hat{z})$ that an agent with money is approached by the money issuer at the beginning of the period and offered to exchange his unit of money into one unit of a commodity which provides a flow utility $(1 - \beta)\gamma$, where $\beta\gamma > c$. As a result, if z is sufficiently large, an agent is willing to produce in exchange for money, irrespective of the behavior of the other agents. Thus very large values of z capture instances where the money issuer is fully committed with keeping the value of money (e.g., the gold standard system). In states $z < -\hat{z}$, if an agent chooses to bring money into a trading sector, there is a probability $p(z) = |z - \hat{z}|/(1 + |z - \hat{z}|)$ that his money will be taxed away by the money issuer.⁶ In particular, very small values

⁴ <http://www.reuters.com/article/2011/01/05>.

⁵ <http://www.forbes.com/sites/nathanlewis/2013/03/14>.

⁶ The taxation of money is a natural way to capture the effects of inflation in an environment with indivisible money.

of z capture a scenario of hyperinflation, in which money is essentially worthless as a medium of exchange. Under this interpretation of $\gamma(z)$, we have $\gamma(z) = p(z)\gamma$ for $z > \hat{z}$ and $\gamma(z) = 0$ for $z < -\hat{z}$.⁷

2.3. Benchmark case: fiat money

We initially consider the scenario where $\hat{z} = \infty$ and money is fiat. In this case, the random variable z is a sunspot, with no impact on payoffs. We are interested in the behavior of an agent without money, who is asked to produce in exchange for money. The decision on whether to offer money in exchange for a desirable good is trivial in our model, since the most an agent can obtain with one unit of money is one unit of good, and all desirable goods provide the same utility u .

In principle, the behavior of an agent may depend on the history of states and, consequently, on the agent's belief about the behavior of all other agents in all future periods, and after every possible history of states. However, a natural approach if money is fiat is to look at equilibria where agents' behavior do not depend on the history of states. One such equilibrium is autarky, where each agent believes that all other agents will never accept money. Under some conditions on parameters, another equilibrium is money, in which each agent believes that all other agents will always accept money. Let $\mathcal{V}_{1,z}$ be the value function of an agent with money in state z if he believes that all other agents will always accept money, and let $\mathcal{V}_{0,z}$ be defined in a similar way for an agent without money. We have

$$\mathcal{V}_{1,z} = m\beta E_z \mathcal{V}_1 + (1-m)(u + \beta E_z \mathcal{V}_0), \quad (1)$$

and

$$\mathcal{V}_{0,z} = m[\sigma(-c + \beta E_z \mathcal{V}_1) + (1-\sigma)\beta \mathcal{V}_0] + (1-m)\beta E_z \mathcal{V}_0, \quad (2)$$

where $\beta E_z \mathcal{V}_1$ is the expected payoff of carrying one unit of money into the next period, $\beta E_z \mathcal{V}_0$ is the expected payoff of carrying zero units of money into the next period, and $\sigma \in [0, 1]$ is the probability that the agent accepts money. Consider the expression for $\mathcal{V}_{1,z}$. An agent with money goes to the sector that trades the good he likes. In this sector, there is a probability m that he meets another agent with money and no trade happens. There is also a probability $(1-m)$ that he meets an agent without money, in which case they trade, the agent obtains utility u , and moves to the next period without money. A similar reasoning holds for an agent without money.

Assume that $\sigma = 1$. Then, for all z ,

$$\mathcal{V}_{1,z} - \mathcal{V}_{0,z} = (1-m)u + mc. \quad (3)$$

It is indeed optimal to always accept money as long as $-c + \beta \mathcal{V}_{1,z} \geq \beta \mathcal{V}_{0,z}$, i.e.,

$$\beta[(1-m)u + mc] \geq c, \quad (4)$$

which is henceforth assumed. Thus, if money is fiat and (4) holds, there always exist an autarkic equilibrium and a monetary equilibrium. Kiyotaki and Wright (1993) show that there also exists an equilibrium where agents are indifferent between accepting and not accepting money. A similar equilibrium exists here.⁸

2.4. General case

We now consider the case where \hat{z} is finite, so there are states where money carries some intrinsic utility. We are particularly interested in the behavior of agents who are asked to produce in exchange for money in states where money yields no intrinsic utility.

Let $\varphi(t)$ denote the probability that any state $z' > z$ is reached at time $s+t$ and not before, conditional on the agent being in state z in period s . Since Δz_t follows a continuous probability distribution that is independent of the current state z and the period s , $\varphi(t)$ does not depend on z or s . Proposition 1 summarizes our main result.

Proposition 1. *Let $\hat{z} \in \mathbb{R}$. Generically, there exists a unique rationalizable equilibrium in the model. Agents play according to a threshold z^* such that an agent produces in exchange for money if and only if $z \geq z^*$. If*

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) [(1-m)u + mc] > c, \quad (5)$$

then $z^* < -\hat{z}$. If the inequality is reversed, $z^* > \hat{z}$.

⁷ Note that there is no intrinsic disutility associated with bringing money to the market in states $z < -\hat{z}$. Indeed, under this interpretation of $\gamma(z)$, what makes money useless when z is sufficiently small is not the possibility that money produces some negative intrinsic utility, but the fact that money will be taxed away with probability near one. As it will become clear in what follows, what is important is that there exist states where not to accept money is a strictly dominant action, and this is the case in states in which money is taxed away almost certainly. We choose to capture the costs of carrying money in our environment with the negative intrinsic utility $\gamma(z)$ for ease of exposition. Appendix 1 available online formalizes this point.

⁸ These are not the only equilibria in our environment if money is fiat. For instance, there exists an equilibrium in which agents accept money in all states $z \notin \mathbb{Q}$, where \mathbb{Q} is the set of rational numbers, as long as the past history of states only includes states which do not belong to \mathbb{Q} . If, at some point, the current state $z \in \mathbb{Q}$, money is never accepted from that point on. To check that this is an equilibrium, simply note that \mathbb{Q} is a set of measure zero within the real numbers. Thus, how agents coordinate their behavior in states $z \in \mathbb{Q}$ only matters after one such state is reached for the first time. Clearly, infinitely many other equilibria can be constructed in a similar way.

Proposition 1 shows that in the region $[-\hat{z}, \hat{z}]$ agents will always coordinate in the use of money if (5) holds, and will never coordinate in the use of money otherwise. The proof of **Proposition 1** is organized in steps. In **Lemma 2**, we show that agents' decisions on whether to produce in exchange for money are strategic complements.

Lemma 2 (Strategic complementarity). *Consider two scenarios: in case A, agents produce in exchange for money in states $z \in Z_A$; in case B, agents produce in exchange for money in states $z \in Z_B$. If $Z_B \subset Z_A$, then the net benefit of producing in exchange for money in case A is at least as large as in case B.*

The decision about accepting money is actually a decision between an action and an option. The agent can accept money now, which entails a cost c but puts her in the position of consuming the desirable good at the first time the economy reaches a state where money is accepted by others. Alternatively, the agent can reject money now and wait for another opportunity. In doing so, she saves in effort cost, because the present value of future effort cost is lower than c , but she might miss on an opportunity to spend money quickly. Both the value of accepting money now and the value of the option of accepting money later are increasing in the likelihood that money will be accepted by the other agents. **Lemma 2** shows that an increase in the likelihood that money will be accepted by the other agents has a stronger effect on the value of accepting money now than in the value of the option of accepting money later.

Lemma 2 implies that if it is optimal to accept money in state z under the belief that all other agents accept money if and only if $z' \geq z$, then it is optimal to accept money in state z under the belief that money is accepted in states $z' \geq z$, regardless of the belief about how agents behave in states $z' < z$. This result will be instrumental for our proof. It will ensure that when considering an agent's belief as to how others behave, there is no loss in generality in restricting attention to beliefs that others follow symmetric cut-off strategies, i.e., strategies in which an agent accepts money if and only if the current state is above than or equal to some threshold state.

Lemma 3 shows that if an agent believes that all the other agents are following a cut-off strategy at state z , then the best response is also a cut-off strategy. This best response is unique and we denote the corresponding cut-off state by $Z(z)$.

Lemma 3 (Optimality of cut-off strategy). *For every $z \in \mathbb{R}$, there exists a unique $Z(z)$ such that if an agent believes that all the other agents follow a cut-off strategy at z , then she produces in exchange for money if and only if $z' \geq Z(z)$.*

The intuition for **Lemma 3** is similar to the intuition for **Lemma 2**. Again, there is a trade-off between producing in exchange for money and consuming the desirable good at the first time the economy reaches a state where money is accepted, and postponing the effort cost. If an agent believes that all other agents are following a cut-off strategy at some state z , and z' is much larger than z , it makes sense to produce in exchange for money now, as the probability that money will be spent quickly is relatively high. In contrast, if z' is much smaller than z , it makes more sense to wait for a future opportunity of producing in exchange for money, since there will probably be other opportunities of doing so before the agent can spend it. **Lemma 3** shows that this intuition applies more generally: if others are following a cut-off strategy at some state z , the payoff of producing in exchange for money for an agent is increasing in z' . Hence, the best response to cut-off strategies is also a cut-off strategy.

Lemma 4 uses **Lemma 2** and **3** to show that if an agent believes that all the other agents are following a cut-off strategy at state z , then her expected payoff employing the same cut-off strategy is sufficient to determine the optimal behavior in state z , even though in general that is not her optimal strategy.

Let $\tilde{V}_1(z)$ be the value of holding one unit of money at the end of period s , under the assumption that all agents, including the agent herself, follows a cut-off strategy at state z . $\tilde{V}_0(z)$ is similarly defined for an agent holding zero units of money. We have the following result.

Lemma 4 (Optimal behavior). *$\tilde{V}_1(z) - \tilde{V}_0(z)$ is strictly increasing in $z \notin [-\hat{z}, \hat{z}]$ and constant in $z \in [-\hat{z}, \hat{z}]$. Moreover, in any state $z \in [-\hat{z}, \hat{z}]$, if $\tilde{V}_1(z) - \tilde{V}_0(z) > c$, it is strictly optimal to produce in exchange for money. Otherwise, it is optimal not to produce in exchange for money.*

The fact that $\tilde{V}_1(z) - \tilde{V}_0(z)$ is independent of z if $z \in [-\hat{z}, \hat{z}]$ is simple to demonstrate. Indeed, for all $z \in [-\hat{z}, \hat{z}]$,

$$\tilde{V}_1(z) - \tilde{V}_0(z) = \sum_{t=1}^{\infty} \beta^t \varphi(t) \left[\int_z^{\infty} (V_{1,z'} - V_{0,z'}) dF(z'|t) \right]. \quad (6)$$

In words, conditional on the belief that money is only accepted in states $z' > z$, the net expected payoff of holding money is given by the discounted probability that the first time the economy reaches some state $z' > z$ after period s happens in period $s+t$ (which is given by $\beta^t \varphi(t)$) multiplied by the net expected value of having money in state z' at the beginning of the period, that is, the expected value of $V_{1,z'} - V_{0,z'}$. The cdf of z' is given by $F(z'|t)$, i.e., the probability that the state of the economy is below or equal to z' conditional on the event that the first time the economy reaches some state $z' > z$ after period s happens in period $s+t$. Now, if an agent follows a cut-off strategy at z and if she believes that all other agents follow the same cut-off strategy, the value of having money in some state $z' > z$ is given by

$$V_{1,z'} = m\beta E_z V_1 + (1-m)(u + \beta E_z V_0), \quad (7)$$

while the value of not having money in some state $z' > z$ is

$$V_{0,z'} = m(-c + \beta E_z V_1) + (1-m)\beta E_z V_0. \quad (8)$$

The expected values $E_z V_0$ and $E_z V_1$ are complicated objects but our analysis is vastly simplified because $V_{1,z'} - V_{0,z'}$ does not depend on them.⁹ Using (7) and (8),

$$V_{1,z'} - V_{0,z'} = (1 - m)u + mc. \quad (9)$$

In (7), we used the assumption $(1 - m)u + mc > \gamma$. Combined with $z \in [-\hat{z}, \hat{z}]$, it implies that an agent always prefers to bring money to the market in states $z' > z$ instead of keeping money throughout the period in order to enjoy its intrinsic positive flow utility. In turn, in (8), we used the fact that in states below z , an agent prefers to keep money throughout the period, instead of attempting to use it in some trading post.

We can then substitute (9) into (6) and obtain

$$\tilde{V}_1(z) - \tilde{V}_0(z) = \sum_{t=1}^{\infty} \beta^t \varphi(t) [(1 - m)u + mc]. \quad (10)$$

Since Δz_t follows a random walk, $\varphi(t)$ is independent of z . As a result, the right-hand side of (10) does not depend on z .

Consider now the result that $\tilde{V}_1(z) - \tilde{V}_0(z)$ is strictly increasing in z for $z \notin [-\hat{z}, \hat{z}]$. The intuition for the proof runs as follows. When $z < -\hat{z}$, agents can spend money in the interval $[z, -\hat{z})$, but it is costly to do so. A lower value of z implies both a larger region where spending money is costly and a higher cost of spending money close to z , hence it reduces the incentives for accepting it. In turn, when $z > \hat{z}$, agents cannot spend money in the interval $(\hat{z}, z]$, but they obtain a positive intrinsic flow utility from having money in that region. A larger z implies both a larger region where money brings some intrinsic utility (in case it cannot be spent) and a higher intrinsic flow utility close to z .

The key result of Lemma 4 is that, if an agent believes that all other agents are following a cut-off strategy at state $z \in [-\hat{z}, \hat{z}]$, then in order to know whether she finds it optimal to accept money in state z , all we need to know is what the agent would choose if she was following a cut-off strategy at state z . Indeed, Lemma 4 shows that if an agent strictly prefers to accept money in state z , then she must be following a threshold strategy at some state $Z(z) < z$. The proof is by contradiction. If $Z(z) \geq z$, then $\tilde{V}_1(z) - \tilde{V}_0(z) > c$ would imply $\tilde{V}_1[Z(z)] - \tilde{V}_0[Z(z)] > c$ since $\tilde{V}_1(z) - \tilde{V}_0(z)$ is weakly increasing in z everywhere. However, using Lemma 2, $\tilde{V}_1[Z(z)] - \tilde{V}_0[Z(z)] > c$ implies the agent would strictly prefer to accept money in state $Z(z)$ if others were following a cutoff strategy around $z \leq Z(z)$. This contradicts the definition of $Z(z)$ as the cut-off state. Intuitively, compared to accepting money in state z , incentives for accepting money at state $Z(z) \geq z$ can only increase if all other agents are following a cut-off strategy at z .

We are now ready to prove Proposition 1. The proof uses an induction argument, where at each step strictly dominated strategies are eliminated. We start with the problem of an agent on whether to produce in exchange for money if the current state is large. Owing to the increasing positive intrinsic utility of money in states $z > \hat{z}$, and the fact that $\lim_{z \rightarrow \infty} \gamma(z) = \gamma > c/\beta$, to produce in exchange for money is a strictly dominant action if z is large enough. Indeed, as $z \rightarrow \infty$, agents strictly prefer to produce in exchange for money even if they believe that they will never be able to spend it. Hence there exists some Z_H such that accepting money is a strictly dominant action whenever $z > Z_H$. Consider now the problem of an agent on whether to produce in exchange for money in the current state is $z = Z_H$. This agent knows that all the other agents always accept money if $z > Z_H$. Lemma 2 then implies that the worst possible scenario for an agent that accepts money occurs when all others are playing a cut-off strategy at state Z_H . In this case, a lower bound on $\tilde{V}_1(z) - \tilde{V}_0(z)$ is given by (10) in all states $z > -\hat{z}$, as it does not include the intrinsic utility of holding money when money is not accepted as a medium of exchange. We can then use Lemma 4 to conclude that, if (5) holds, an agent finds it strictly optimal to accept money in state $z = Z_H$. Now, by continuity on z , there exists some ϵ such that the agent strictly prefers to accept money in state $Z_H - \epsilon$. In consequence, once not-accepting money has been ruled out for all $z \geq Z_H$, accepting money becomes a strictly dominant action for an agent in any state $z > Z_H - \epsilon$. We can then apply Lemma 4 once more to conclude that, if (5) holds, an agent finds it strictly optimal to accept money in state $z = Z_H - \epsilon$. Proceeding this way, and using the fact $\tilde{V}_1(z) - \tilde{V}_0(z)$ is strictly increasing in z in states $z < -\hat{z}$, we eventually reach a state $z^* < -\hat{z}$ such that $\tilde{V}_1(z^*) - \tilde{V}_0(z^*) = c$.

An analogous reasoning applies if we consider the problem of an agent on whether to produce in exchange for money if the state z is small. Since bringing money to a trading post implies an intrinsic disutility $\gamma(z)$, and since $\lim_{z \rightarrow -\infty} \gamma(z) = \phi < -u$, as $z \rightarrow -\infty$, accepting money in exchange for production becomes a strictly dominated action. Thus there exists some Z_L such that not accepting money is a strictly dominant action whenever $z < Z_L$. Consider now the problem of an agent in state $z = Z_L$. Note that (10) is an upper bound for $\tilde{V}_1(z^*) - \tilde{V}_0(z^*)$ in all states $z < -\hat{z}$ as it does not include the cost associated with bringing money to a trading post. We can then use Lemma 4 to conclude that, if the reverse of (5) holds, an agent finds it strictly optimal not to accept money in state $z = Z_L$. Now, by continuity on z , there exists some ϵ such that the agent strictly prefers not to accept money in state $Z_L + \epsilon$. In consequence, once accepting money has been ruled out for all $z \leq Z_L$, to not accept money becomes a strictly dominant action for an agent in any state $z < Z_L + \epsilon$. Proceeding this way, and using the fact $\tilde{V}_1(z) - \tilde{V}_0(z)$ is strictly increasing in z in states $z > \hat{z}$, we eventually reach a state $z^* > \hat{z}$ such that $\tilde{V}_1(z^*) - \tilde{V}_0(z^*) = c$.

In consequence, except in a measure zero set where the expression in (10) happens to be equal to c , the iterative process of eliminating strictly dominated strategies will end up at a unique point $z^* < -\hat{z}$ or at $z^* > \hat{z}$.

⁹ The assumption that agents meet at random in a given market plays a key role here. Since everyone meets an agent with money with probability m , potential buyers (who have money) will use money with probability $1 - m$, and potential sellers (who have no money) will receive money in exchange for their good with probability m . In consequence, the continuation values for potential buyers and potential sellers are the same.

3. Interpreting the result

In what follows, we look at (5) in more detail in order to examine how changes in the primitives of the environment affect society's ability to coordinate in the use of money and we examine the role played by the intrinsic value of money.

3.1. Coordination and the primitives of the environment

An increase in the time discount factor can be interpreted as a reduction in the time interval between two consecutive periods. In our environment, since there is one meeting per period, this amounts to an increase in the arrival rate of trading opportunities.¹⁰ Thus, in the discussion below, we think of an increase in the time discount factor as capturing an increase in the frequency of trade meetings.

To better understand the role of the primitives, it is useful to rewrite (4) and (5) as

$$\beta \left[(1-m) \frac{u}{c} + m \right] \geq 1, \quad (11)$$

and

$$\lambda \beta \left[(1-m) \frac{u}{c} + m \right] > 1, \quad (12)$$

where

$$\lambda = \sum_{t=1}^{\infty} \beta^{t-1} \varphi(t). \quad (13)$$

The key difference between (11) and (12) is that (11) takes as given that all other agents will coordinate in the use of money, and examines how changes in primitives affect the incentives of an *individual* agent to produce in exchange for money; while (12) examines how changes in primitives affect *society's* ability to actually coordinate in the use of money. The contribution of our paper lies in uncovering the latter effect. First, a simple inspection of (11) and (12) makes it transparent that these effects are somewhat linked, and the same forces which create incentives to produce in exchange for money under the belief that all other agents accept money; also help to coordinate in its use. However, a more careful analysis unveils a distinctive role to the frequency of trade meetings. In order to make this point clear, let

$$\mathcal{L} \left(\frac{u}{c}, m \right) = (1-m) \frac{u}{c} + m, \quad (14)$$

and compare

$$\left. \frac{\partial \beta}{\partial \mathcal{L} \left(\frac{u}{c}, m \right)} \right|_{\beta \mathcal{L} \left(\frac{u}{c}, m \right) = 1} = - \frac{\beta}{\mathcal{L} \left(\frac{u}{c}, m \right)} \quad (15)$$

with

$$\left. \frac{\partial \beta}{\partial \mathcal{L} \left(\frac{u}{c}, m \right)} \right|_{\lambda \beta \mathcal{L} \left(\frac{u}{c}, m \right) = 1} = - \frac{\beta}{\mathcal{L} \left(\frac{u}{c}, m \right)} \frac{\sum_{t=1}^{\infty} \beta^{t-1} \varphi(t)}{\sum_{t=1}^{\infty} t \beta^{t-1} \varphi(t)}. \quad (16)$$

The derivative in (15) shows by how much β has to increase when there is a unit decrease in $(1-m)u/c+m$, in order for an agent to be willing to produce in exchange for money, under the belief that all other agents accept money. In turn, the derivative in (16) shows by how much β has to increase when there is a unit decrease in $(1-m)u/c+m$, in order for all agents to coordinate in the use of money. The key implication is that, if $(1-m)u/c+m$ decreases by one unit, the required increase in β for coordination reasons is smaller than the one required to sustain an agent's individual incentive to produce in exchange for money. In other words, an increase in the frequency of trade meetings, more so than an increase in u/c or a decrease in m , critically helps agents coordinate in the use of money. The intuition runs as follows. If an agent expects to meet many partners in a short period of time, he is willing to accept money even if he believes that a large number of his future partners will not accept money. That affects beliefs of all other agents: they know others will be willing to produce in exchange for money, even if they hold relatively pessimistic beliefs. Consequently, pessimistic beliefs cannot be an equilibrium.

It is particularly interesting to look at the extreme case where β converges to one. To do so, consider first the scenario where the stochastic process that governs Δz is symmetric, that is, $E(\Delta z) = 0$. In this case, since the probability that the economy will eventually reach a state to the right of the current state is equal to one (that is, $\sum_{t=1}^{\infty} \varphi(t) = 1$) it must be that λ

¹⁰ Alternatively, we could consider a setting in which β is fixed, but agents are randomly and anonymously matched in pairs $n \geq 1$ times in each period. An increase in n would then amount to an increase in the arrival rate of trading opportunities. We obtain the same results with this alternative specification, but since the agent's decision on whether to accept money depends on the state of the economy, when there is more than one meeting in a period, we need to assume that the state of the economy changes across meetings and not across periods. This way, if β is the period discount factor and there are n meetings in a period, then $\beta^{1/n}$ is the discount factor in between meetings. The analysis then is exactly the same as in Section 2.4, with $\beta^{1/n}$ replacing β . As n goes to ∞ , this discount factor goes to one.

converges to one as β converges to one. This means that if one restricts attention to the region of parameters where money is an equilibrium in the benchmark model of Section 2.3, then as $\beta \rightarrow 1$, money is always the unique equilibrium in the model of Section 2.4.

Finally, it is possible to construct an example where the probability of ever getting to the region where holding money is a strictly dominant action is arbitrarily small and still, as β goes to 1, λ converges to one. Let $E(\Delta z) = \eta$ for some $\eta < 0$. Since Δz_t follows a continuous probability distribution, as η approaches zero and β goes to one, λ converges to one. Now, for any $\eta < 0$ there exists a large enough \hat{z} so that the probability of ever reaching states $z > \hat{z}$ is arbitrarily small, and the probability of reaching states $z < -\hat{z}$ is equal to one. Therefore, as β goes to one, money is the unique equilibrium in the general model in the region where it is an equilibrium in the benchmark model, despite the probability that money will ever acquire a positive intrinsic utility being arbitrarily small and the probability that it will eventually acquire a negative intrinsic utility being equal to one.

3.2. The role of the intrinsic value of money

The condition in (5) holds for any $\hat{z} \in \mathbb{R}$. In particular, \hat{z} can be as large as we want, and it is in this sense that we think of states where money acquires a negative or a positive intrinsic utility as being faraway states. Moreover, an implication of (5) is that coordination in the use of money in states where money does not provide any intrinsic utility does not depend on $\gamma(z)$. Indeed, when $z \in [-\hat{z}, \hat{z}]$, the possible values of $\gamma(z)$ do not enter an individual agent's problem – even though the intrinsic positive or negative utility of money is important to establish the existence of dominant regions. From the perspective of an agent, the key force driving the coordination in the use of money is the belief on its acceptability as a medium of exchange, and not that it may acquire an intrinsic utility.¹¹

The result of a unique rationalizable equilibrium relies on the existence of both dominance regions. However, if the assumption of negative intrinsic utility of money for $z < -\hat{z}$ is removed, money is still the unique equilibrium as long as (5) holds. The only difference is that now there is a region of parameters where money and autarky can be an equilibrium when $z \in [-\hat{z}, \hat{z}]$.

Perturbations to the process for Δz do not significantly affect the equilibrium conditions, as long as the set of probabilities $\varphi(t)$ of reaching nearby states where money is accepted as a medium of exchange does not change much. In particular, the set of probabilities $\varphi(t)$ are very similar if $E(\Delta z) = 0$ or if $E(\Delta z) = \eta$, where η is a very small (negative or positive) number. Thus, by setting η very small and negative and \hat{z} sufficiently large, we can make the probability that money will have a negative intrinsic utility in the long run arbitrarily close to one, without significantly affecting the condition for money being accepted in all states where it has no intrinsic utility. Thus coordination in the use of money does not come from simply working backwards from some distant future where money acquires positive intrinsic utility with a sufficiently high probability.¹²

The assumption that Δz follows a random walk makes the problem identical at every state z . That simplifies the analysis, but implies that in the long run, irrespective of the value of \hat{z} , the economy will usually be at states outside the $[-\hat{z}, \hat{z}]$ interval. However, a small modification of the random walk process could rule out this outcome without significantly affecting our results. Consider a process such that $E(\Delta z) = -\eta$ for any $z > 0$ and $E(\Delta z) = \eta$ for any $z < 0$. For η sufficiently small, the set of probabilities of reaching a nearby state in the following periods would not be substantially affected, and thus the condition for a unique monetary equilibrium would be very similar to (5). We can then make sure that the economy will rarely be outside of the $[-\hat{z}, \hat{z}]$ interval by choosing a large enough \hat{z} .¹³

We have considered the case of a continuous state space but our results can be extended to a discrete state space, i.e., the case where the set of states is given by \mathbb{Z} . We present this case in the online Appendix 2. The key difference is that, in the discrete case, if the economy is in some state z in the current period, there is a positive probability that it will be in the same state after a couple of periods. This gives rise to an intermediate region of parameters where multiple equilibria exist. In this intermediate region, the possibility of the economy being exactly at the same state in the future might allow agents to coordinate on arbitrary beliefs. However, except for this multiplicity region, results are analogous to the continuous case. Moreover, as the frequency of trade meetings, captured here by the agents' discount factor, goes to infinity, the multiple-equilibrium region disappears.

4. Final remarks

The notion of essentiality, by emphasizing that the societal benefits of money come from the fact that it achieves desirable allocations, helps identify models in which the use of money is justified on grounds of efficiency (Wallace, 2001).

¹¹ If we increased the intrinsic utility of money by assuming that $\gamma > (1-m)u + mc$, agents would not spend money in some faraway states. Lacking the option of accepting money in some future states, agents could choose to accept (but not spend) money now. That would make the problem more complicated, since the decision of the agent with money would not be trivial anymore, without necessarily adding much insight into the problem.

¹² In turn, by setting η very small and positive and \hat{z} sufficiently large, we can make the probability that money will have a positive intrinsic utility in the long-run arbitrarily close to one, without significantly affecting the condition for money not to be accepted in all states where it has no intrinsic utility. The sheer fact that money will acquire a positive intrinsic utility does not ensure that agents will coordinate in its use.

¹³ For positive values of η , there would be a region of parameters where agents would accept money if and only if $z > 0$, but the measure of this region would converge to zero in the limit as η approaches zero.

However, it does not say much about how agents actually end up coordinating in the use of money, and how do the primitives of the environment impact such coordination. We think of our paper as a step in this direction.

We have chosen to present our analysis in a search model of money along the lines of Kiyotaki and Wright (1993), a choice that is mainly driven by tractability reasons. This choice though comes at a cost as their environment is special in some dimensions, particularly the indivisibility of money and the indivisibility of goods – assumptions that have been relaxed in subsequent papers such as Trejos and Wright (1995), Shi (1995, 1997) and Lagos and Wright (2005). We leave the extension to these settings for future work. Since the environment in Kiyotaki and Wright (1993) combines key elements that matter for the problem of coordinating in the use of money in a tractable manner, we see it as a natural starting point.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at <http://dx.doi.org/10.1016/j.jmoneco.2014.01.009>.

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